

following is a remarkable theorem on this part of the subject. The positive pedal of a bicircular quartic is the inverse of its negative pedal if the centre of one of its circles of inversion be taken as origin.

Chapter VII. is the last. It contains the theory of reciprocating curves from their tangential and intrinsic equation. Thus, if $\rho=f(\phi)$ be the tangential equation, its reciprocal is in polar equations

$$\rho = \frac{k^2}{f(\phi) \sin \phi}.$$

Again, if $s=f(\phi)$ be the intrinsic equation of a curve, the polar equation of its reciprocal is

$$\frac{k^2}{\rho} = \left(\frac{d^2}{d\phi^2} + 1 \right)^{-1} f'(\phi).$$

This chapter contains also the theory of parallel curves. The following is a remarkable property of these curves:—Every focus of any order of the original curve is a focus of the next highest order of the parallel curve.

The last problem discussed in the paper is the rectification of bicircular quartics by elliptic integrals, and the method can be extended to sphero-quartics. This problem, so far as the author is aware, is now solved for the first time.

The paper is enriched by the addition of a very important annex by Professor Cayley.

IV. "Addition on the Bicircular Quartic." By A. CAYLEY, LL.D., F.R.S., Sadlerian Professor of Mathematics in the University of Cambridge. Received January 24, 1877.

(Abstract.)

Prof. Casey communicated to me the MS. of the foregoing memoir, and he has permitted me to make to it the present addition, containing further developments on the theory of the bicircular quartic.

Starting from his theory of the fourfold generation of the curve, Prof. Casey shows that there exist series of inscribed quadrilaterals, ABCD, whereof the sides AB, BC, CD, DA pass through the centres of the four circles of inversion respectively; or (as it is convenient to express it) the pairs of points (A, B), (B, C), (C, D), (D, A) belong to the four modes of generation respectively, and may be regarded as depending upon certain parameters (his $\theta, \theta', \theta'', \theta'''$, or say) $\omega_1, \omega_2, \omega_3, \omega$ respectively, any three of these being in fact functions of the fourth. Considering a given quadrilateral ABCD, and giving to it an infinitesimal variation, we have four infinitesimal arcs, AA', BB', CC', DD'; these are in fact differential expressions, AA' and BB' of the form $M_1 d\omega_1$, BB' and CC' of the form $M_2 d\omega_2$, CC' and DD' of the form $M_3 d\omega_3$, DD' and AA' of the form $M d\omega$;

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or, what is the same thing, AA' is expressible in the two forms $Md\omega$ and $M_1d\omega_1$, BB' in the two forms $M_1d\omega_1$ and $M_2d\omega_2$ &c., the identity of the two expressions for the same arc, of course, depending on the relation between the two parameters. But any such monomial expression $Md\omega$ of an arc AA' would be of a complicated form, not obviously reducible to elliptic functions. Casey does not obtain them at all; but he finds geometrically monomial expressions for the differences and sum $BB' - AA'$, $CC' - BB'$, $DD' + CC'$, $DD' - AA'$ (they cannot be all of them differences), and thence a quadrinomial expression $AA' = N_1d\omega_1 + N_2d\omega_2 + N_3d\omega_3 + N_4d\omega_4$ (his $ds' = \rho d\theta + \rho' d\theta' + \rho'' d\theta'' + \rho''' d\theta'''$), and that without any explicit consideration of the relations which connect the parameters.

I propose to complete the analytical theory by establishing the monomial equations $AA' = Md\omega = M_1d\omega_1$, &c., and the relations between the parameters $\omega, \omega_1, \omega_2, \omega_3$, which belong to an inscribed quadrilateral $ABCD$, so as to show what the process really is by which we pass from the monomial form to a quadrinomial form $AA' = N_1d\omega + N_2d\omega_1 + N_3d\omega_2 + N_4d\omega_3$, $= dS$, wherein each term is separately expressible as the differential of an elliptic integral, and to further develop the theory of the transformation to elliptic integrals.

V. "On the Influence of Height in the Atmosphere on the Diurnal Variation of the Earth's Magnetic Force." By J. A. BROUN, F.R.S. Received January 25, 1877.

In a paper in the Society's Transactions on the earth's magnetic intensity at Bombay, Mr. C. Chambers has examined the question of the influence of height on the diurnal inequality of the horizontal force*. Two instruments were observed simultaneously at 0^h 22^m and 2^h 29^m P.M.: one, a bifilar magnetometer, was 6 feet above the ground; the other, a unifilar absolute-intensity instrument, was 38 feet above the ground.

Mr. Chambers has found that, in the interval between the two times specified, the change of horizontal force given by the bifilar magnetometer was rather more than one third (0.37) of the mean diurnal range, and that the absolute intensity instrument showed a change about one fifth less than the bifilar. This difference he does not think instrumental; and he considers that, if true, "it suggests the attribution of a very considerable magnetic influence to the state of the medium intervening between the upper and lower places of observation," &c.

The mean changes of horizontal force between 0^h 22^m and 2^h 29^m P.M. by the two instruments were as follow:—

* "The Absolute Direction and Intensity of the Earth's Magnetic Force at Bombay, &c." By Charles Chambers, F.R.S., Superintendent of the Colaba Observatory. Phil. Trans. 1876, p. 84.